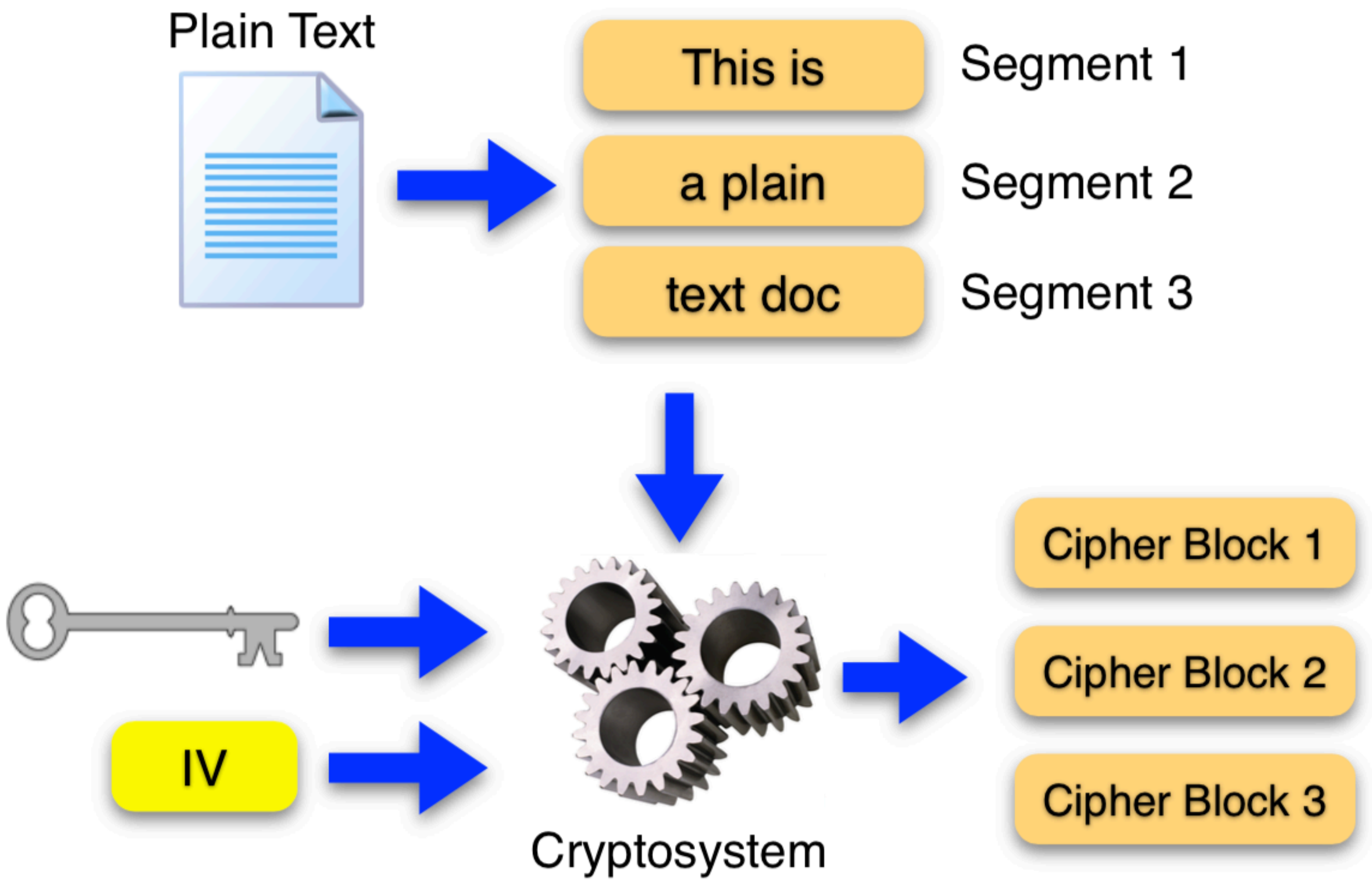
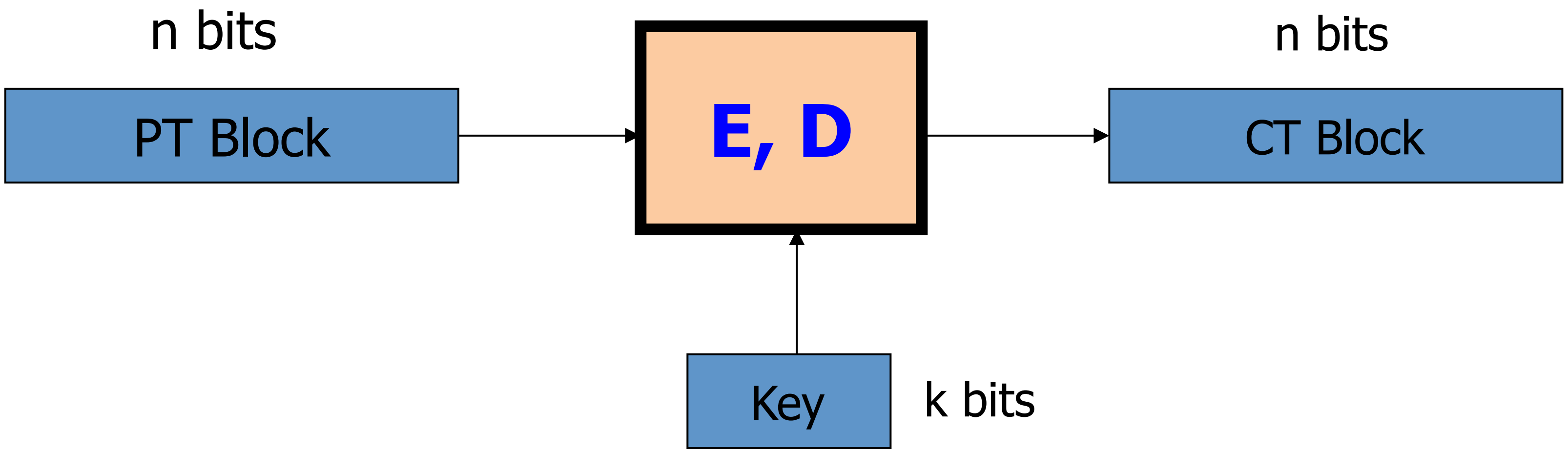
Block Ciphers

Block Cipher



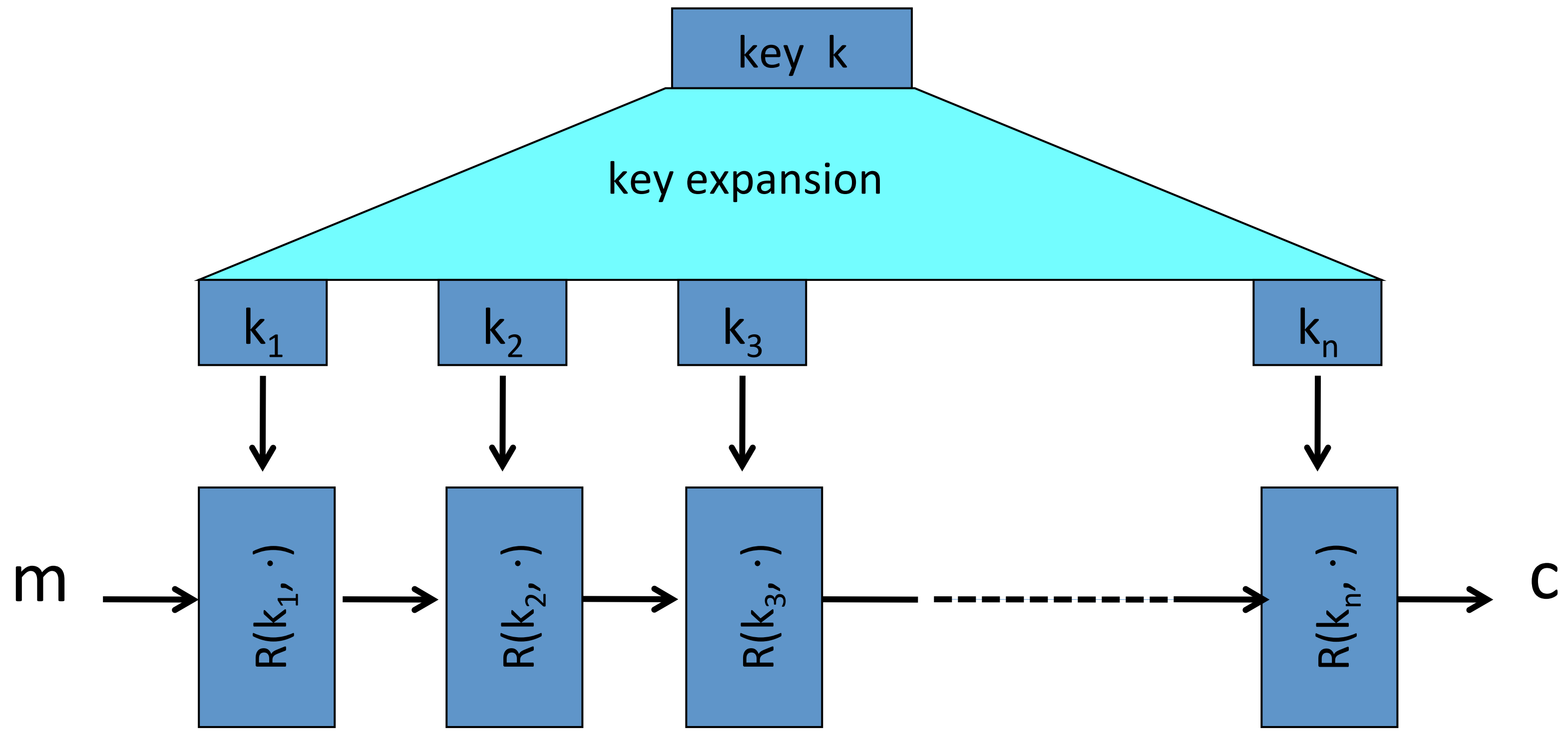
Block ciphers: crypto work horse



Canonical examples:

1. 3DES: n= 64 bits, k = 168 bits
2. AES: n=128 bits, k = 128, 192, 256 bits

Block Ciphers Built by Iteration



R(k,m) is called a round function

for 3DES (n=48), for AES-128 (n=10)

(Iterated) Block Cipher

- Plaintext and ciphertext consist of fixed-sized blocks
- Ciphertext obtained from plaintext by iterating a **round function**
- Input to round function consists of ***key*** and ***output*** of previous round
- Usually implemented in software

Feistel Cipher: Encryption

- **Feistel cipher** is a type of block cipher, not a specific block cipher
- Split plaintext block into left and right halves: $P = (L_0, R_0)$
- For each round $i = 1,2,\ldots,n$, compute

  $L_i = R_{i-1}$

  $R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$

  where F is **round function** and K_i is **subkey**
- Ciphertext: $C = (L_n, R_n)$

Feistel Cipher: Decryption

- Start with ciphertext $C = (L_n, R_n)$
- For each round $i = n, n-1, \ldots, 1$, compute

  $R_{i-1} = L_i$

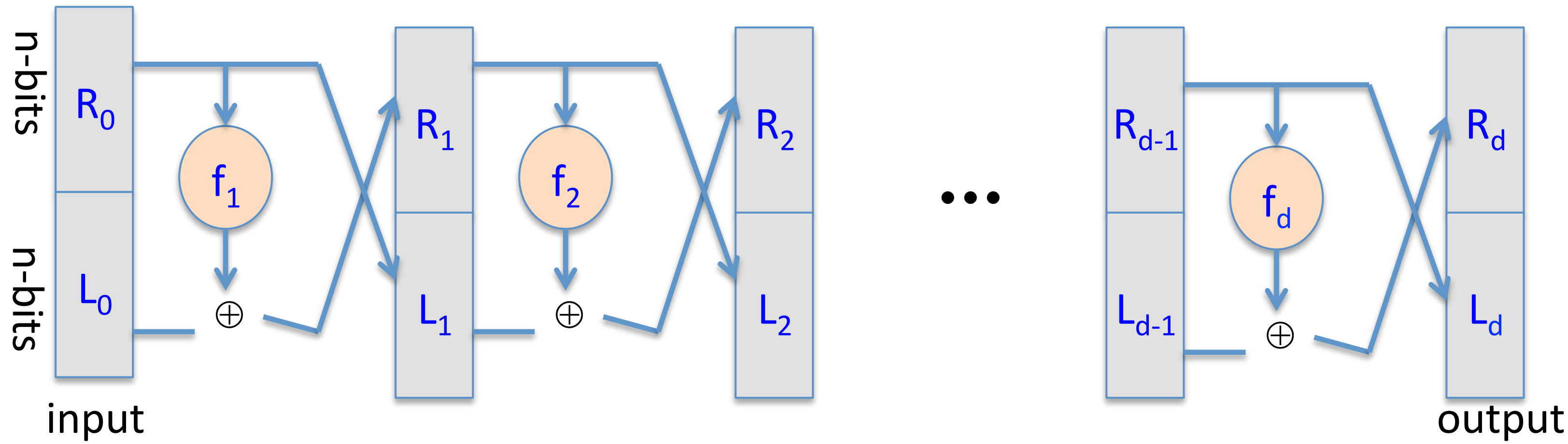
  $L_{i-1} = R_i \oplus F(R_{i-1}, K_i)$

  where F is round function and K_i is subkey
- Plaintext: $P = (L_0, R_0)$
- Formula "works" for any function F
  - But only secure for certain functions F

DES: core idea – Feistel Network

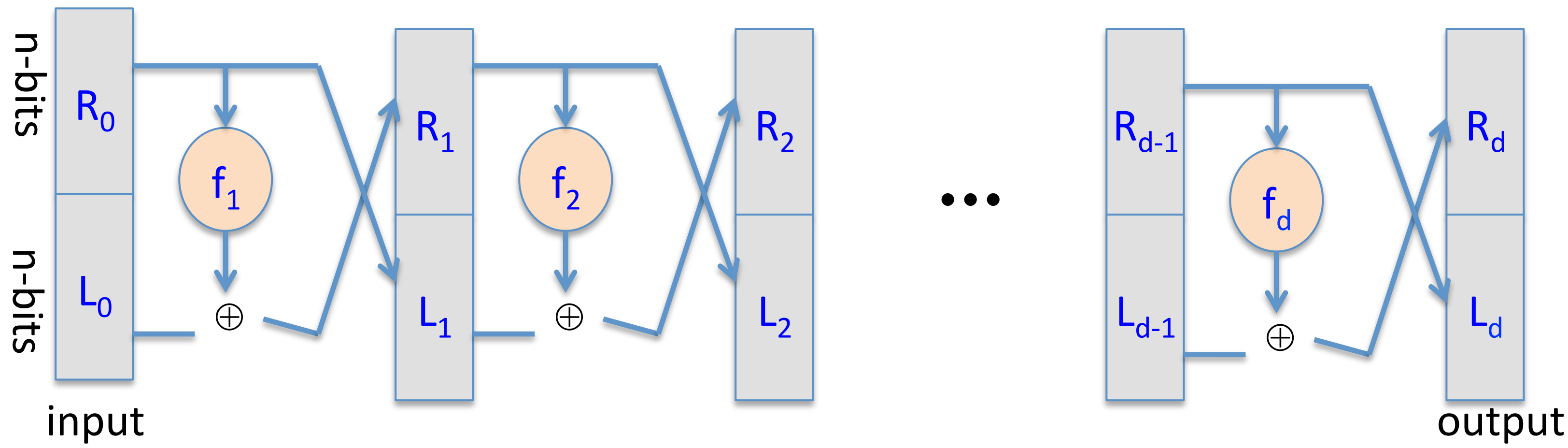
Given functions $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Goal: build invertible function $F: \{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$



In symbols:

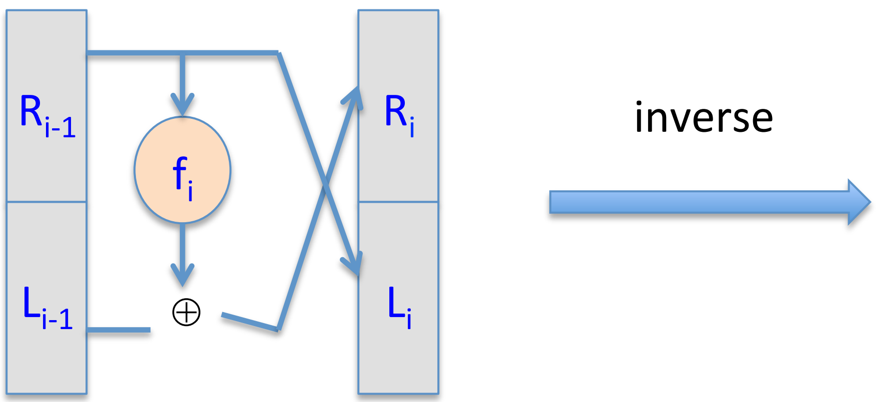
$$\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1} \\ L_i = R_{i-1} \end{cases}$$



Claim: for all $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Feistel network F: $\{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ is invertible

Proof: construct inverse



$R_{i-1} = L_i$

$L_{i-1} =$

Data Encryption Standard

- **DES** developed in 1970's
- Based on IBM's Lucifer cipher
- DES was U.S. government standard
- DES development was controversial
  - NSA secretly involved
  - Design process was secret
  - Key length reduced from 128 to 56 bits
  - Subtle changes to Lucifer algorithm

The Data Encryption Standard (DES)

- Early 1970s: Horst Feistel designs Lucifer at IBM

  key-len = 128 bits ; block-len = 128 bits

- 1973: NBS asks for block cipher proposals.
  IBM submits variant of Lucifer.

- 1976: NBS adopts DES as a federal standard

  key-len = 56 bits ; block-len = 64 bits

- 1997: DES broken by exhaustive search

- 2000: NIST adopts Rijndael as AES to replace DES

Widely deployed in banking (ACH) and commerce

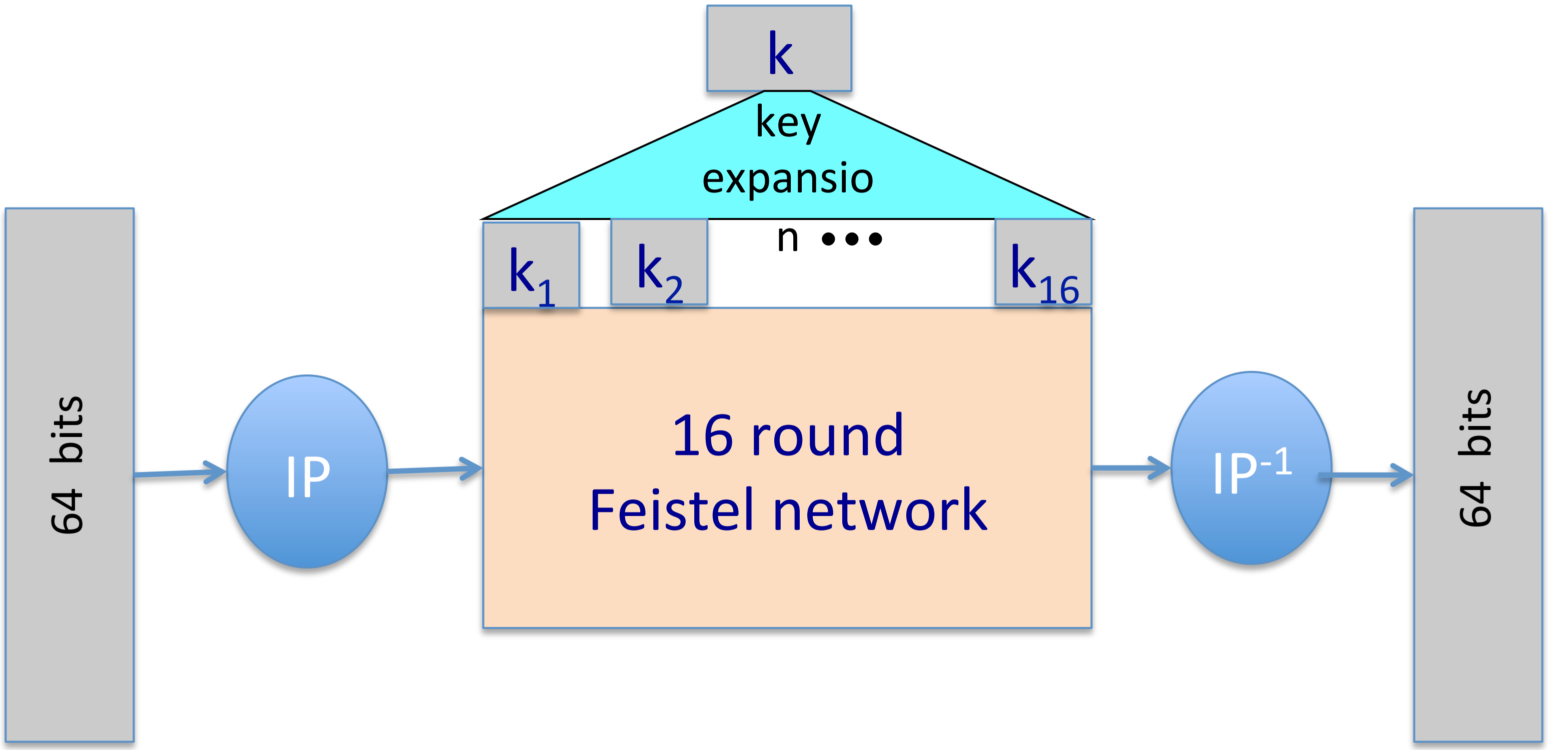
DES Numerology

- DES is a Feistel cipher with…
  - 64 bit block length
  - 56 bit key length
  - 16 rounds
  - 48 bits of key used each round (subkey)
- Each round is simple (for a block cipher)
- Security depends heavily on "S-boxes"
  - Each S-boxes maps 6 bits to 4 bits

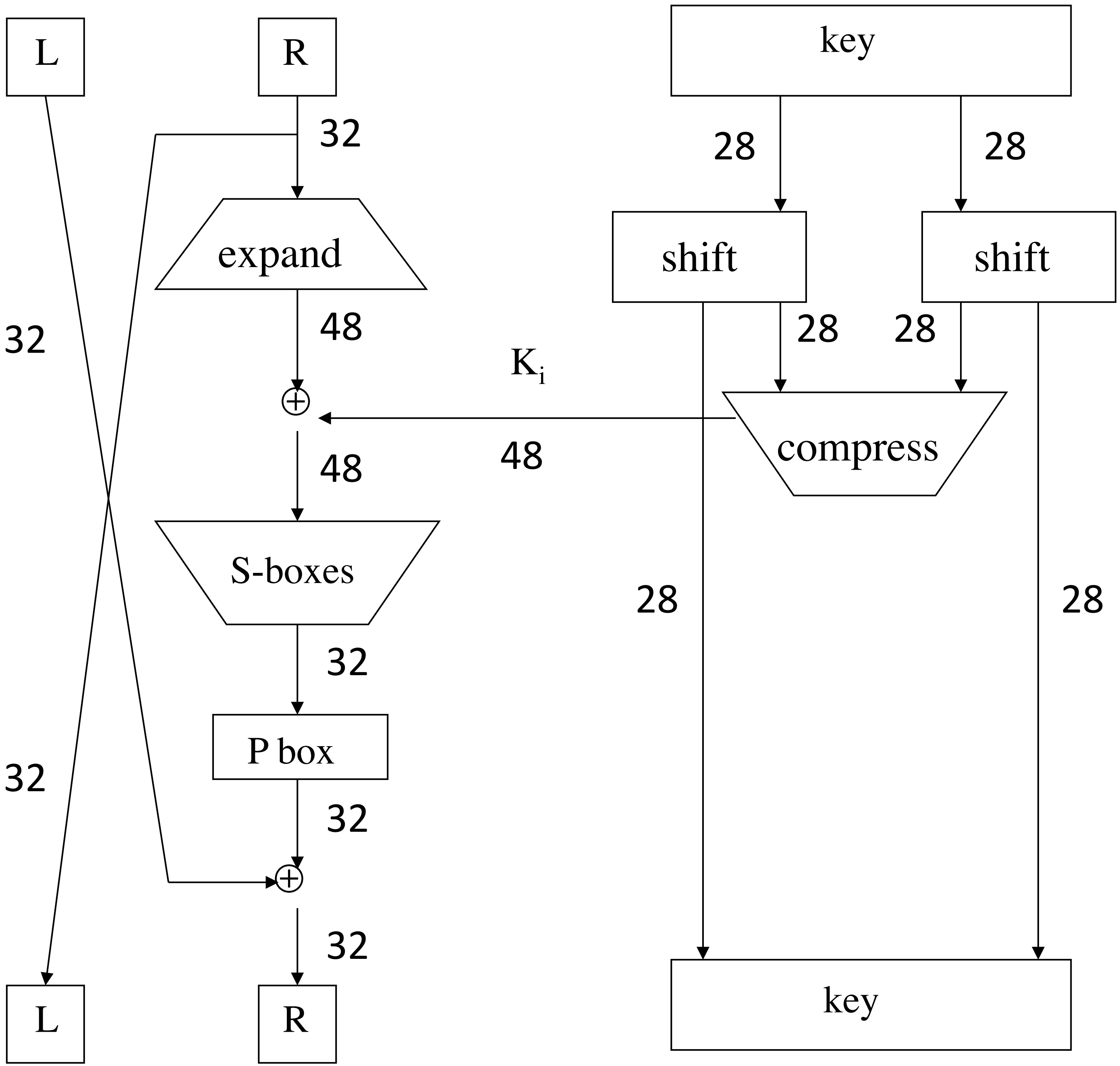
DES: 16 round Feistel network

$f_1, \ldots, f_{16}: \{0,1\}^{32} \longrightarrow \{0,1\}^{32}$, $f_i(x) = \mathbf{F}(k_i, x)$

↑ from key k



To invert, use keys in reverse order



One Round of DES

DES Expansion Permutation

- Input 32 bits

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

- Output 48 bits

31	0	1	2	3	4	3	4	5	6	7	8
7	8	9	10	11	12	11	12	13	14	15	16
15	16	17	18	19	20	19	20	21	22	23	24
23	24	25	26	27	28	27	28	29	30	31	0

DES S-box

- 8 “substitution boxes” or S-boxes
- Each S-box maps 6 bits to 4 bits
- S-box number 1

```
input bits (0,5)
↓                    input bits (1,2,3,4)
   | 0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111
-------------------------------------------------------------------------------------
00 | 1110 0100 1101 0001 0010 1111 1011 1000 0011 1010 0110 1100 0101 1001 0000 0111
01 | 0000 1111 0111 0100 1110 0010 1101 0001 1010 0110 1100 1011 1001 0101 0011 1000
10 | 0100 0001 1110 1000 1101 0110 0010 1011 1111 1100 1001 0111 0011 1010 0101 0000
11 | 1111 1100 1000 0010 0100 1001 0001 0111 0101 1011 0011 1110 1010 0000 0110 1101
```

DES P-box

- Input 32 bits

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

- Output 32 bits

15	6	19	20	28	11	27	16	0	14	22	25	4	17	30	9
1	7	23	13	31	26	2	8	18	12	29	5	21	10	3	24

DES Subkey

- 56 bit DES key, numbered 0,1,2,…,55
- Left half key bits, `LK`

```
49 42 35 28 21 14  7
 0 50 43 36 29 22 15
 8  1 51 44 37 30 23
16  9  2 52 45 38 31
```

- Right half key bits, `RK`

```
55 48 41 34 27 20 13
 6 54 47 40 33 26 19
12  5 53 46 39 32 25
18 11  4 24 17 10  3
```

DES Subkey

- For rounds `i=1,2,...,16`
  - Let LK = (LK circular shift left by r_i)
  - Let RK = (RK circular shift left by r_i)
  - Left half of subkey K_i is of LK bits

```
13 16 10 23  0  4  2 27 14  5 20  9
22 18 11  3 25  7 15  6 26 19 12  1
```

  - Right half of subkey K_i is RK bits

```
12 23  2  8 18 26  1 11 22 16  4 19
15 20 10 27  5 24 17 13 21  7  0  3
```

DES Subkey

- For rounds 1, 2, 9 and 16 the shift r_i is 1, and in all other rounds r_i is 2
- Bits 8,17,21,24 of LK omitted each round
- Bits 6,9,14,25 of RK omitted each round
- **Compression permutation** yields 48 bit subkey K_i from 56 bits of LK and RK
- **Key schedule** generates subkey

DES Last Word (Almost)

- An initial permutation before round 1
- Halves are swapped after last round
- A final permutation (inverse of initial perm) applied to (R_{16},L_{16})
- None of this serves security purpose

Security of DES

- Security depends heavily on S-boxes
  - Everything else in DES is linear
- Thirty+ years of intense analysis has revealed no "back door"
- Attacks, essentially exhaustive key search
- **Inescapable conclusions**
  - Designers of DES knew what they were doing
  - Designers of DES were way ahead of their time

Block Cipher Notation

- P = plaintext block
- C = ciphertext block
- Encrypt P with key K to get ciphertext C
  - $C = E(P, K)$
- Decrypt C with key K to get plaintext P
  - $P = D(C, K)$
- Note: $P = D(E(P, K), K)$ and $C = E(D(C, K), K)$
  - But $P \neq D(E(P, K_1), K_2)$ and $C \neq E(D(C, K_1), K_2)$ when $K_1 \neq K_2$

Triple DES

- Today, 56 bit DES key is too small
  - Exhaustive key search is feasible
- But DES is everywhere, so what to do?
- **Triple DES** or **3DES** (112 bit key)
  - $C = E(D(E(P,K_1),K_2),K_1)$
  - $P = D(E(D(C,K_1),K_2),K_1)$
- Why Encrypt-Decrypt-Encrypt with 2 keys?
  - Backward compatible: $E(D(E(P,K),K),K) = E(P,K)$
  - And 112 bits is enough