

Activity Recognition 3

Classification

Mobile Computing

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2012. 9

Entropy: Formula

- The entropy (in bits) of a discrete random variable M :

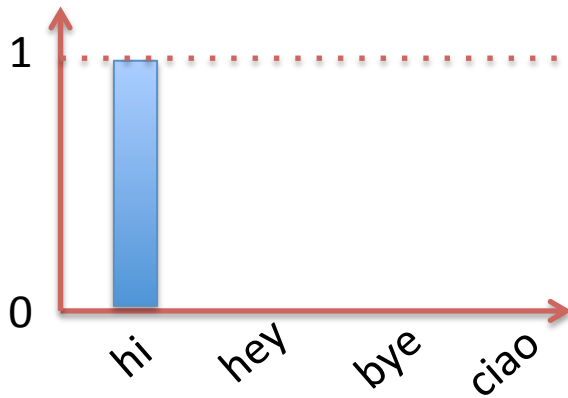
$$\begin{aligned} H(M) &= - \sum_m \Pr(M = m) \log_2 \Pr(M = m) \\ &= - \sum_m p_m \log_2 p_m \\ &= \sum_m p_m \log_2 \frac{1}{p_m} \end{aligned}$$

- Interpretation
 - Average # of bits to express each message
- Maximized when uniform
 - p_m is the same for all messages

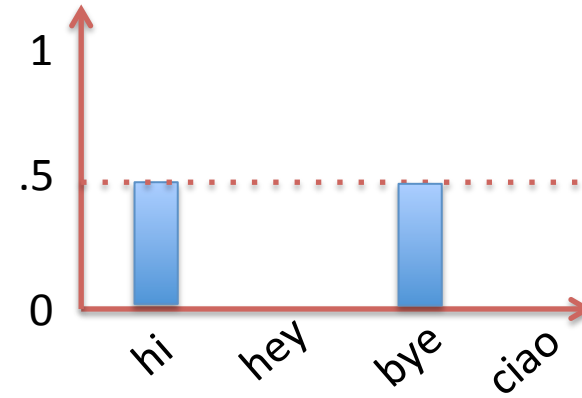
Message Distribution

- Speakers have different message distribution
 - 4 messages possible: “hi”, “hey”, “bye”, “ciao”
 - Alice always says “hi”
 - Bob says only “hi” or “bye” with same probability
 - Cathy says “bye” half the time, and “hi” and “hey” half the time with equal probability
 - David says “hi”, “hey”, “bye”, “ciao” with equal probability

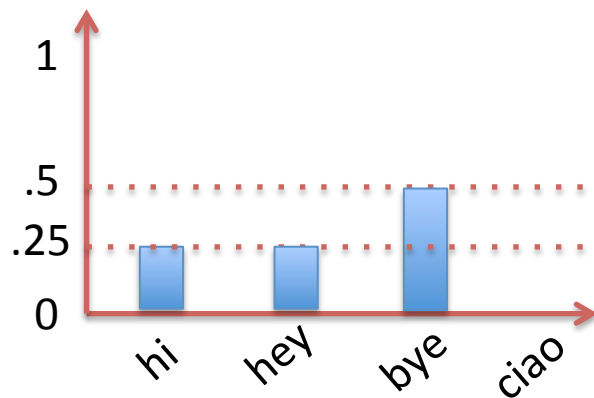
Message Distribution



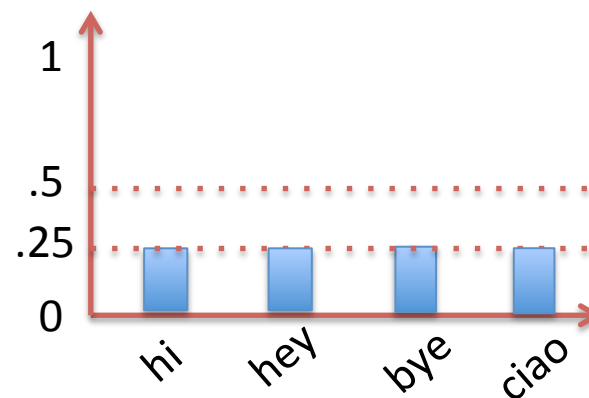
Alice



Bob

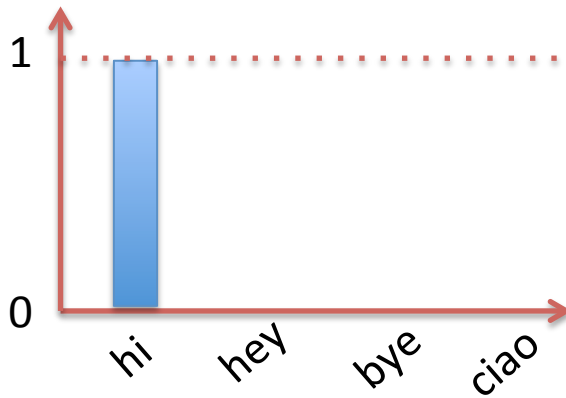


Cathy

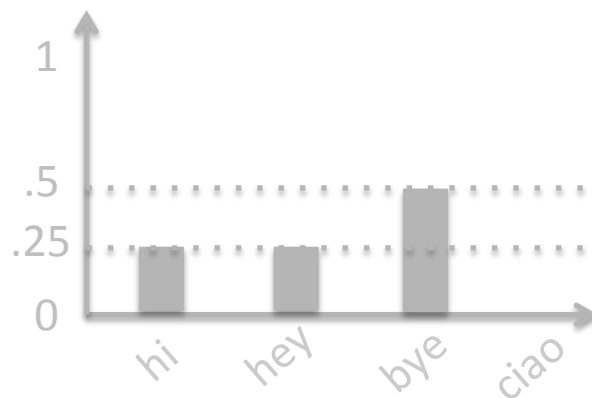


David

Message Distribution



Alice



Cathy

$$\begin{aligned} p_{hi} &= 1 & p_{hey} &= 0 \\ p_{bye} &= 0 & p_{ciao} &= 0 \end{aligned}$$

$$\begin{aligned} H(M) &= p_{hi} \log \frac{1}{p_{hi}} + p_{hey} \log \frac{1}{p_{hey}} \\ &\quad + p_{bye} \log \frac{1}{p_{bye}} + p_{ciao} \log \frac{1}{p_{ciao}} \end{aligned}$$

$$\begin{aligned} H(M) &= 1 \cdot \log \frac{1}{1} + 0 + 0 + 0 \\ &= 0 \end{aligned}$$

David

Message Distribution

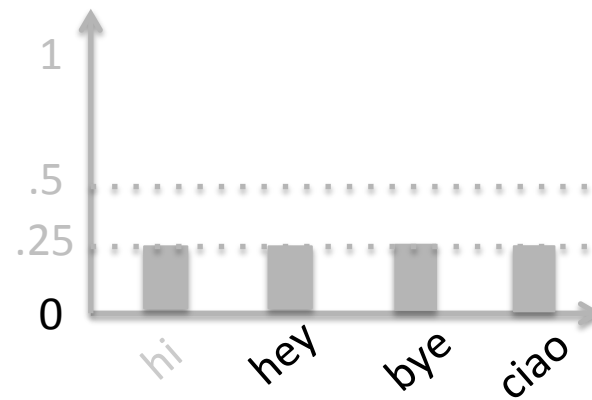
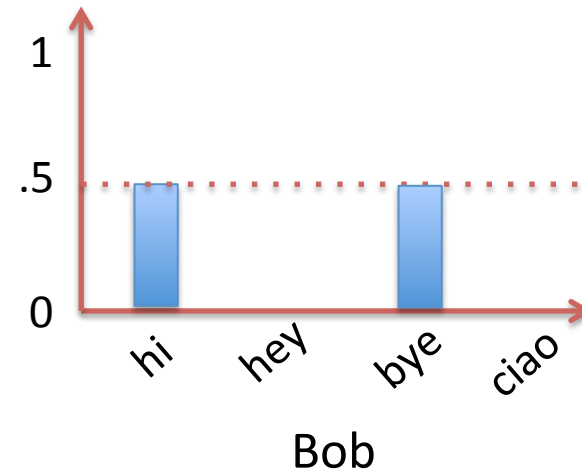
$$p_{hi} = 0.5 \quad p_{hey} = 0$$

$$p_{bye} = 0.5 \quad p_{ciao} = 0$$

$$\begin{aligned} H(M) &= p_{hi} \log \frac{1}{p_{hi}} + p_{hey} \log \frac{1}{p_{hi}} \\ &\quad + p_{bye} \log \frac{1}{p_{bye}} + p_{ciao} \log \frac{1}{p_{ciao}} \end{aligned}$$

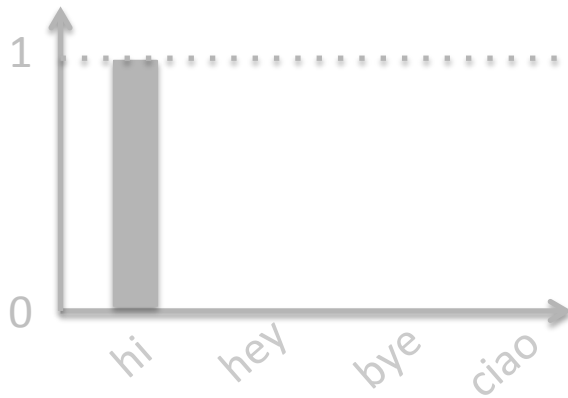
$$\begin{aligned} H(M) &= \frac{1}{2} \cdot \log \frac{1}{\frac{1}{2}} + 0 \\ &\quad + \frac{1}{2} \cdot \log \frac{1}{\frac{1}{2}} + 0 \\ &= \frac{1}{2} + \frac{1}{2} = 1 \end{aligned}$$

Cathy

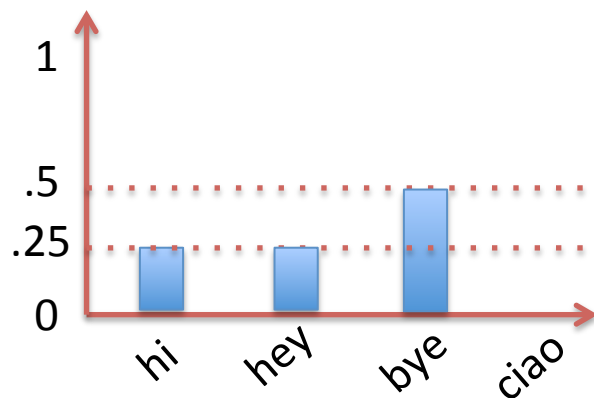


David

Message Distribution



Alice



Cathy

$$p_{hi} = 0.25 \quad p_{hey} = 0.25$$

$$p_{bye} = 0.5 \quad p_{ciao} = 0$$

$$H(M) = p_{hi} \log \frac{1}{p_{hi}} + p_{hey} \log \frac{1}{p_{hi}} \\ + p_{bye} \log \frac{1}{p_{bye}} + p_{ciao} \log \frac{1}{p_{ciao}}$$

$$H(M) = \frac{1}{4} \cdot \log \frac{1}{\frac{1}{4}} + \frac{1}{4} \cdot \log \frac{1}{\frac{1}{4}} \\ + \frac{1}{2} \cdot \log \frac{1}{\frac{1}{2}} + 0 \\ = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1.5$$

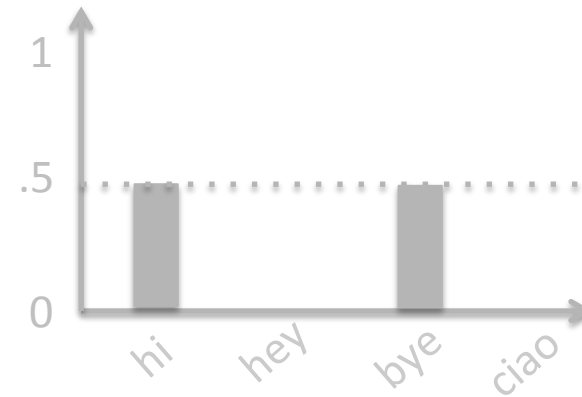
David

Message Distribution

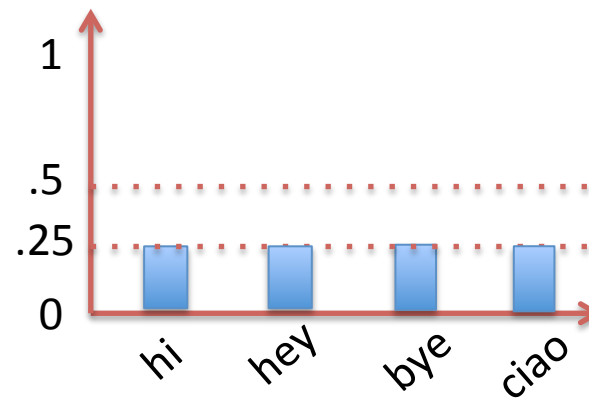
$$\begin{aligned} p_{hi} &= 0.25 & p_{hey} &= 0.25 \\ p_{bye} &= 0.25 & p_{ciao} &= 0.25 \end{aligned}$$

$$\begin{aligned} H(M) &= p_{hi} \log \frac{1}{p_{hi}} + p_{hey} \log \frac{1}{p_{hey}} \\ &\quad + p_{bye} \log \frac{1}{p_{bye}} + p_{ciao} \log \frac{1}{p_{ciao}} \end{aligned}$$

$$\begin{aligned} H(M) &= 4 \times \frac{1}{4} \cdot \log \frac{1}{\frac{1}{4}} \\ &= 2 \end{aligned}$$



Bob



David

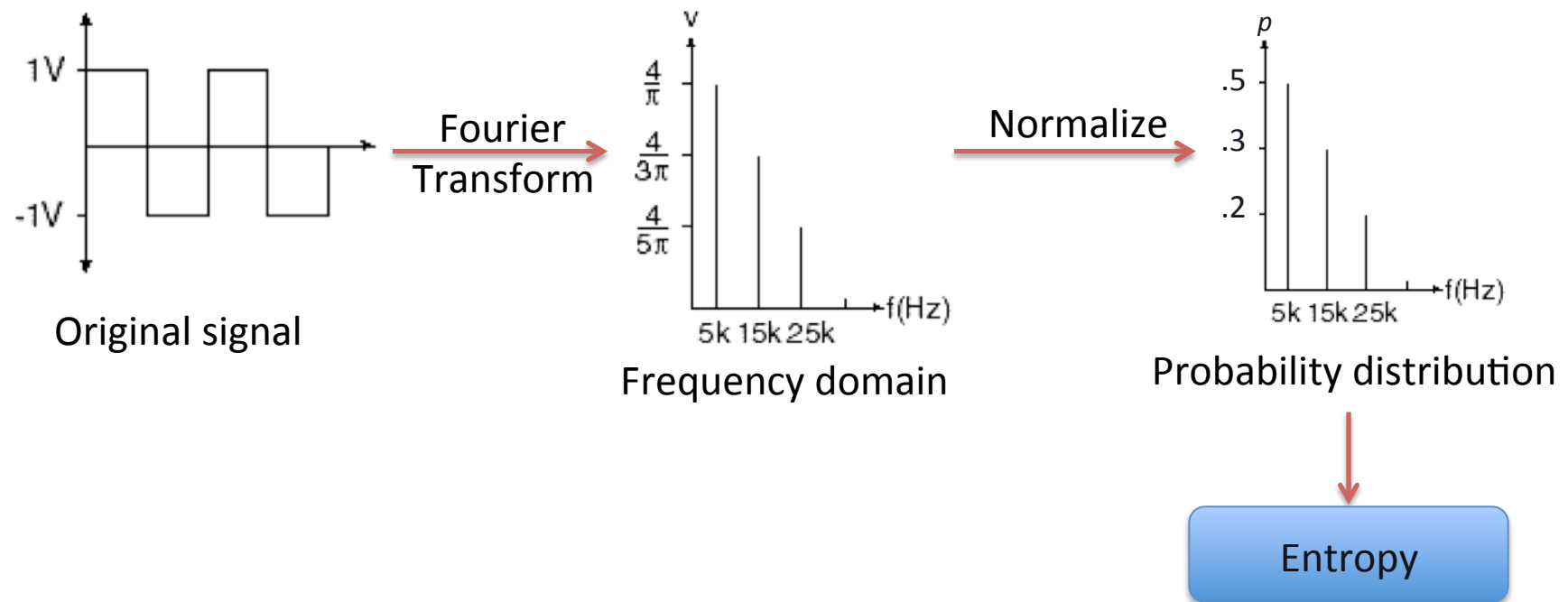
Cathy

Features (revisited)

- Frequency-domain entropy
 - Differentiate between walking and cycling
- What is *frequency domain*?
- What is *Entropy*?
- What is *frequency-domain entropy*?

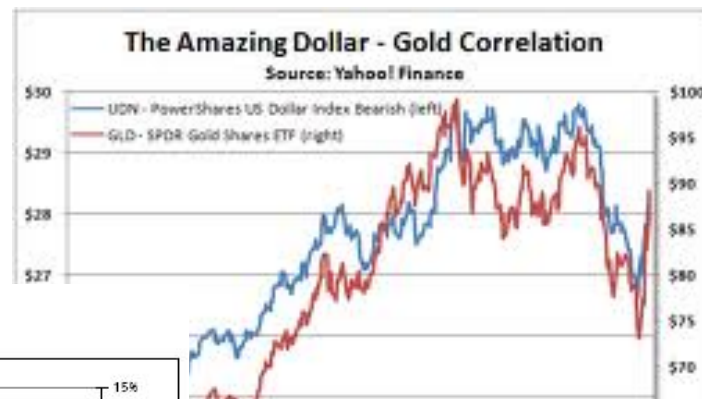
Frequency-domain Entropy

- Given a signal in time-domain, convert to frequency-domain, normalize it, then compute entropy

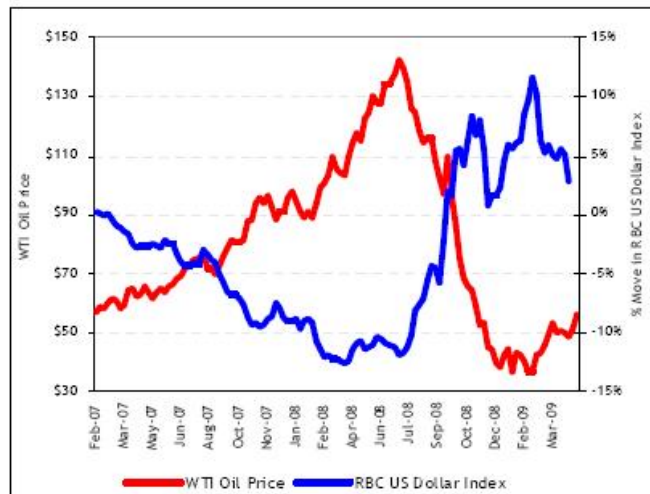


Correlation

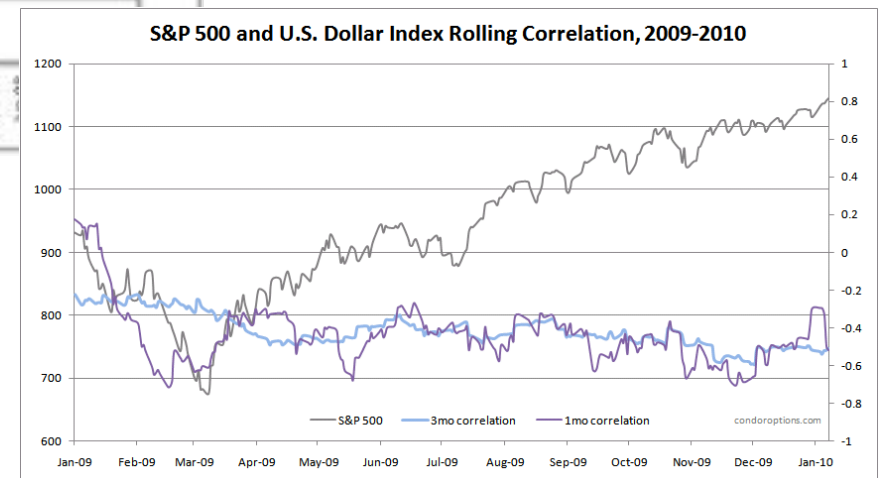
- Degree of dependency between two signals



WTI Crude vs. US Dollar



Source: Bloomberg



Correlation Coefficient

- Given two random variables X, Y , corr-coef is

$$\text{corr}(X, Y) = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y} \in [-1, 1]$$

- Given two series of n measurements x_i and y_i

$$\text{corr}(X, Y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}$$

- Interpretation

- $+1$: perfect dependency
- 0 : no dependency
- -1 : opposite dependency

$$\bar{x} = \frac{\sum_i x_i}{n} \quad \bar{y} = \frac{\sum_i y_i}{n}$$

Feature Set

- X average
- Y average
- Z average
- X variance
- Y variance
- Z variance
- X energy
- Y energy
- Z energy
- X entropy
- Y entropy
- Z entropy
- X, Y correlation
- Y, Z correlation
- Z, X correlation

Feature selection/extraction

- Curse of dimensionality
 - If dimension of features is high, classification becomes difficult
- Solution
 - Reduce the number of features
 - Feature selection: remove less important features
 - Feature extraction: generate smaller # of features

Feature selection

- Given a feature set F , get a subset G of F
- Discarding features that are little helpful for classification
- But, finding the subset is exponentially expensive
 - For example, if $F = \{f_1, f_2, \dots, f_d\}$ (d is F 's dimension), for $m = 1, 2, \dots, d$, the we have to check all subsets of F of size m

Feature selection: Suboptimal Algorithms

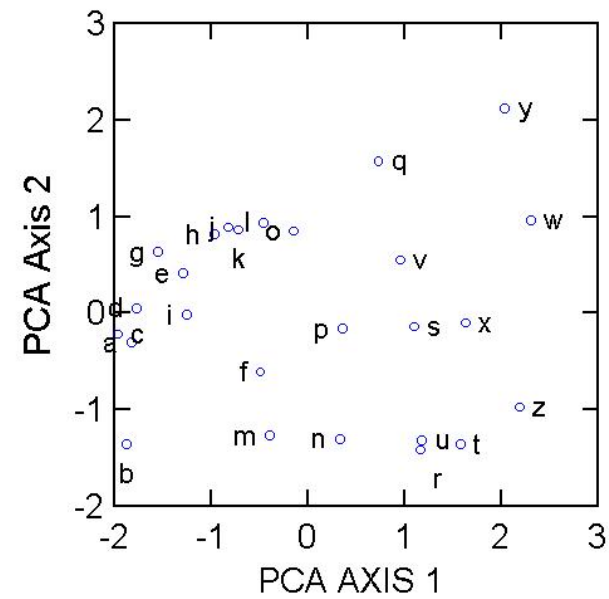
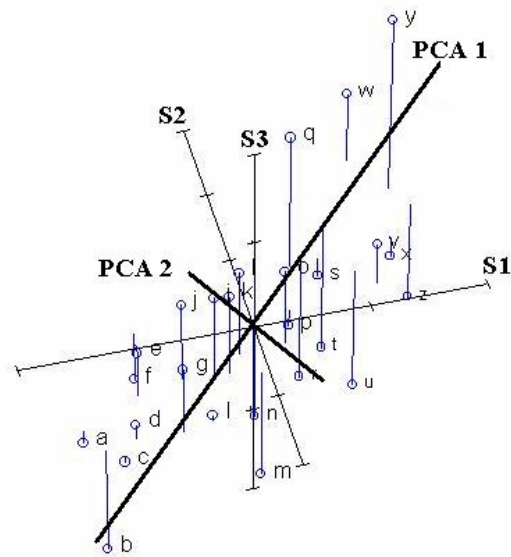
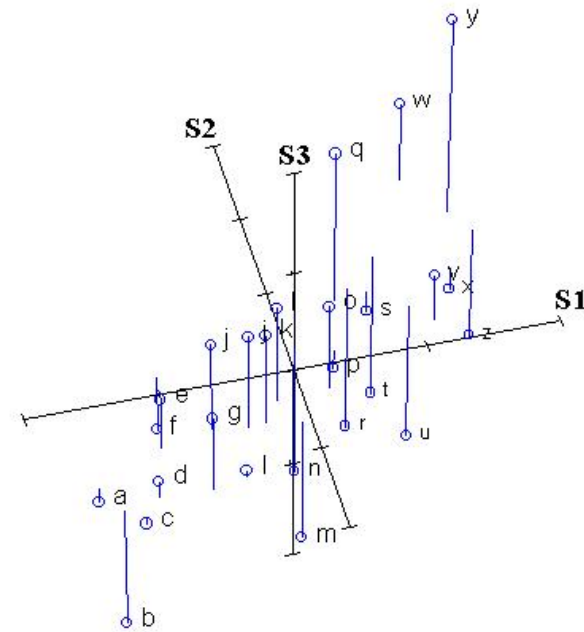
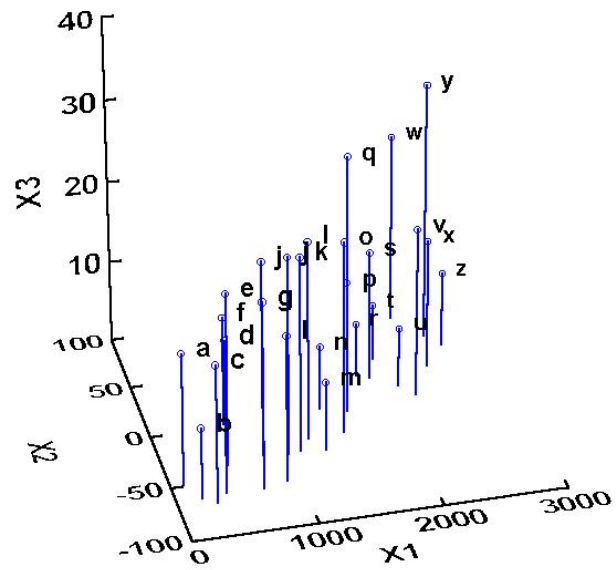
- Use sub-optimal search algorithm
 - Branch-and-bound search
 - Sequential forward/backward search (SFS-SBS)
 - Sequential forward/backward floating search (SFFS-SBFS)
- Sequential search algorithms
 - Iterative procedure
 - Add or remove some features at each step so that the new set leads to a better classification performance, measured by
 - Inter-class distance / intra-class distance
 - Analyze classifier output

Feature Extraction

- Idea: another data representation can be constructed in a subspace (less dimension) while keeping discriminative capability
- Lose physical meaning
- Example algorithms
 - PCA (Principle Component Analysis): transform features into small number of uncorrelated variables
 - ICA (Independent Component Analysis)
- Feature selection and extractions can be used together

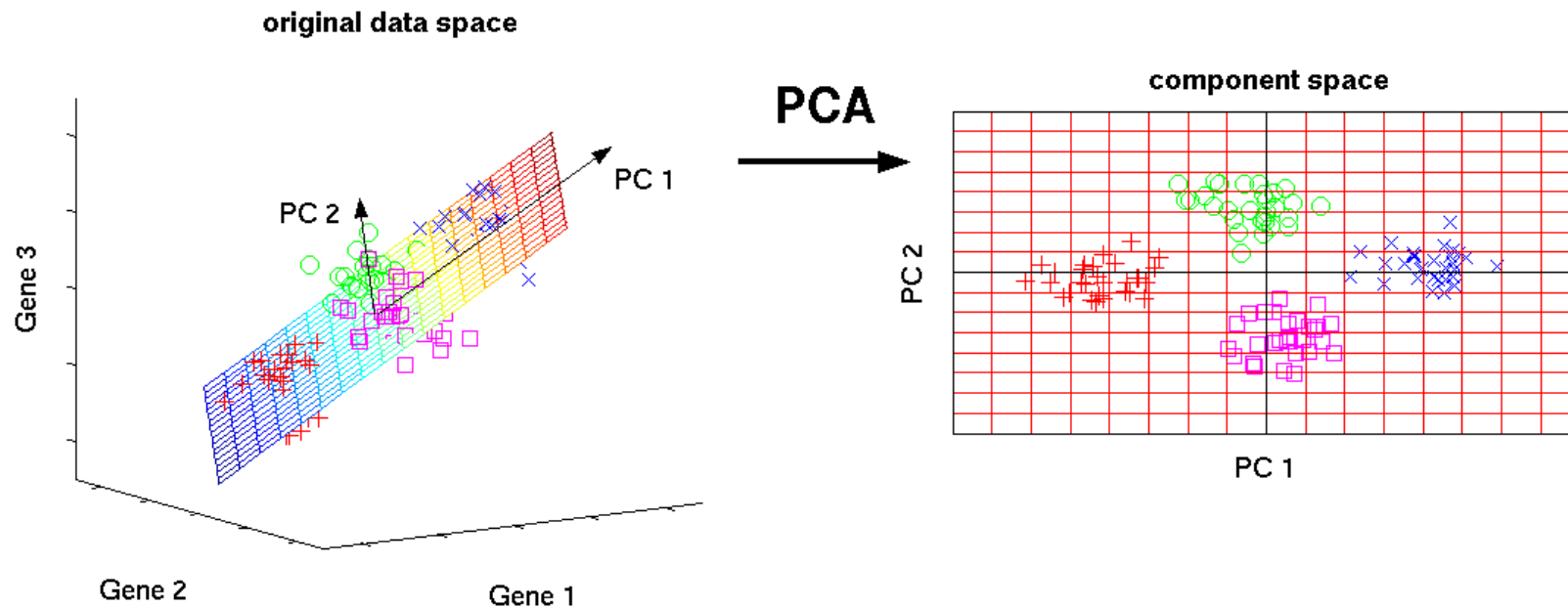
PCA

- How can we reduce dimension?

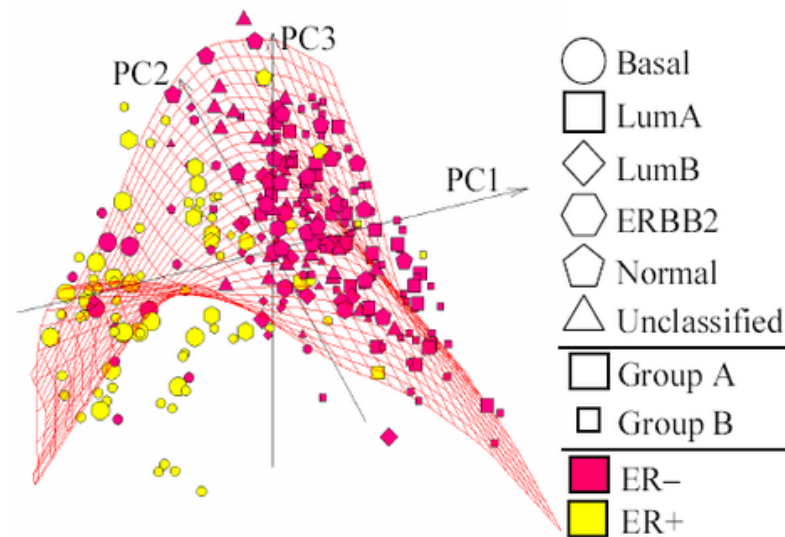


PCA

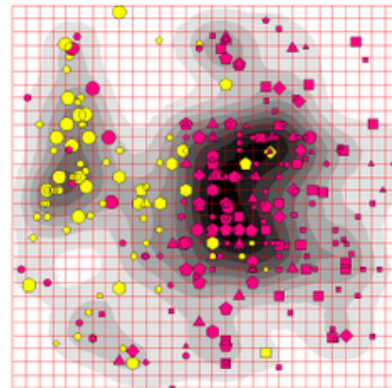
- Identify orthogonal axes with maximum variance of data
 - discard the axis with low variance



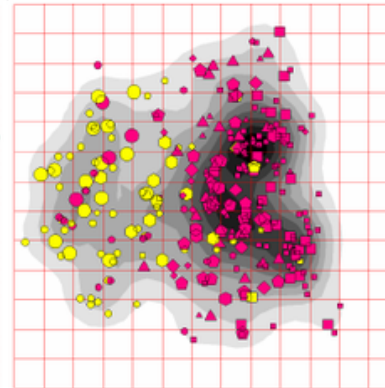
Non-linear PCA



a)



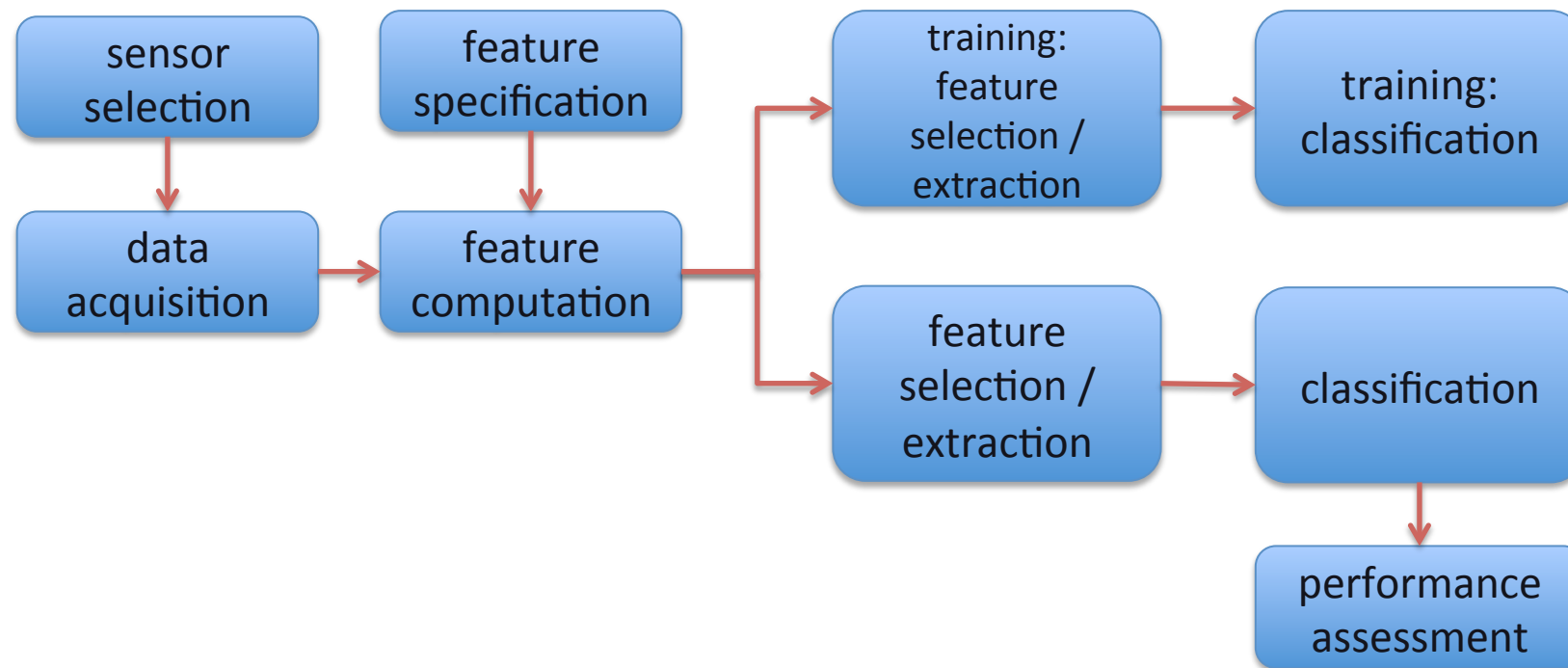
b) ELMAP2D



c) PCA2D

Classification with supervised learning

- Classification: determine the type of activity



Type of Classifiers

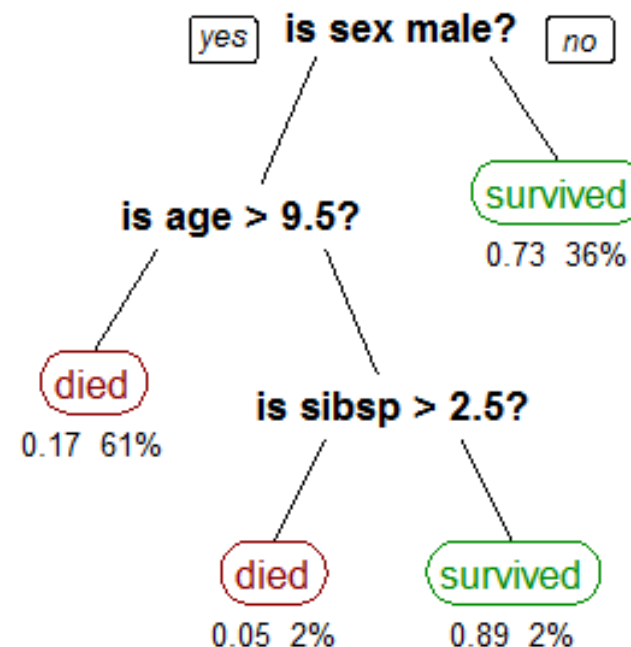
- Supervised & Unsupervised
 - Supervised: class membership of each feature vector is known
 - Unsupervised: Only the number of classes is known
- Single-frame & Sequential
 - Single-frame: Each frame is classified regardless of previous frames
 - Sequential: Each frame is classified in consideration of previous frames

Type of Classifiers

- Probabilistic & Geometric & Template matching
 - Template matching: Based on similarity between data and templates obtained by training or defined by the designer
 - Binary classifier: Descend a binary decision tree from the root to leaves as refining the classification

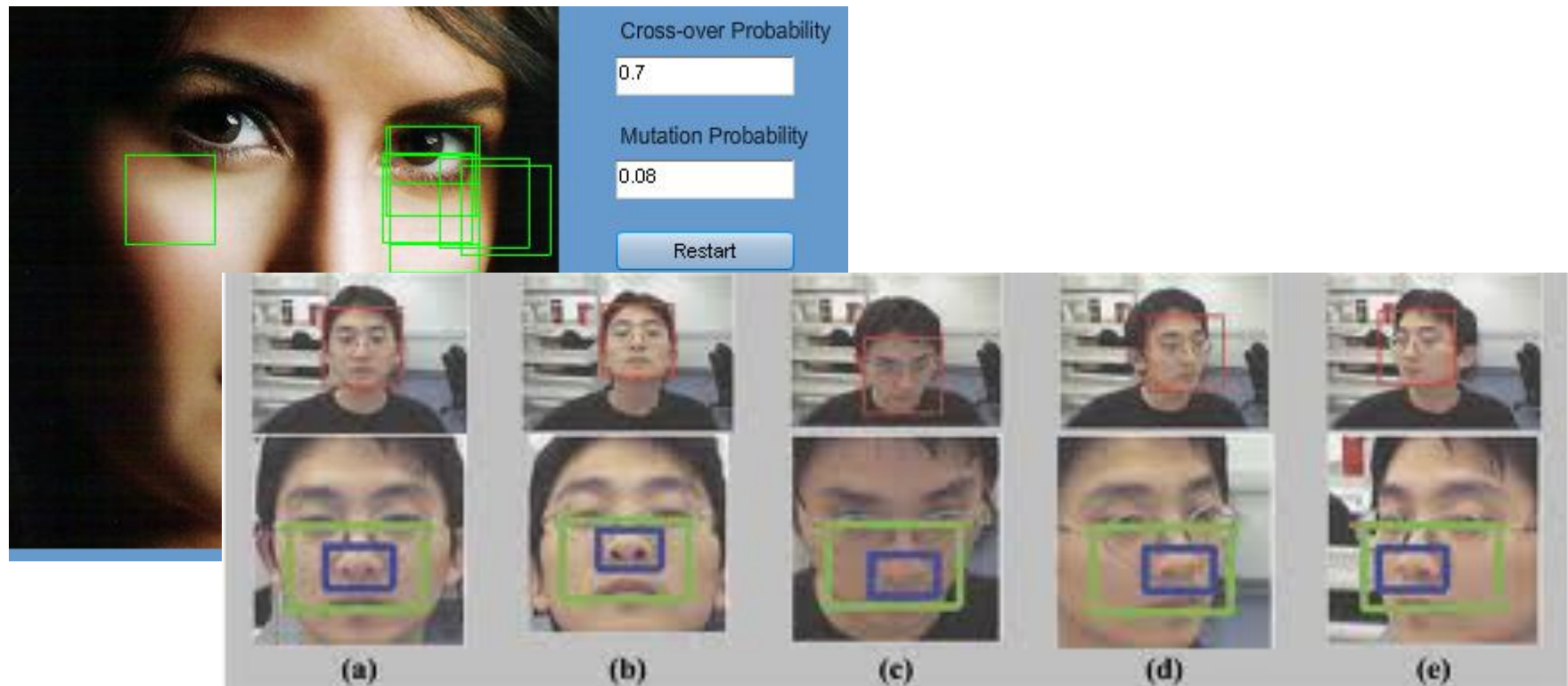
Decision Tree

- “20 Questions”
- Build a tree by dividing the data into two sets recursively, until remains only one class



Template Matching

- Compare test data with well-prepared template, mainly used for image processing



Type of Classifiers

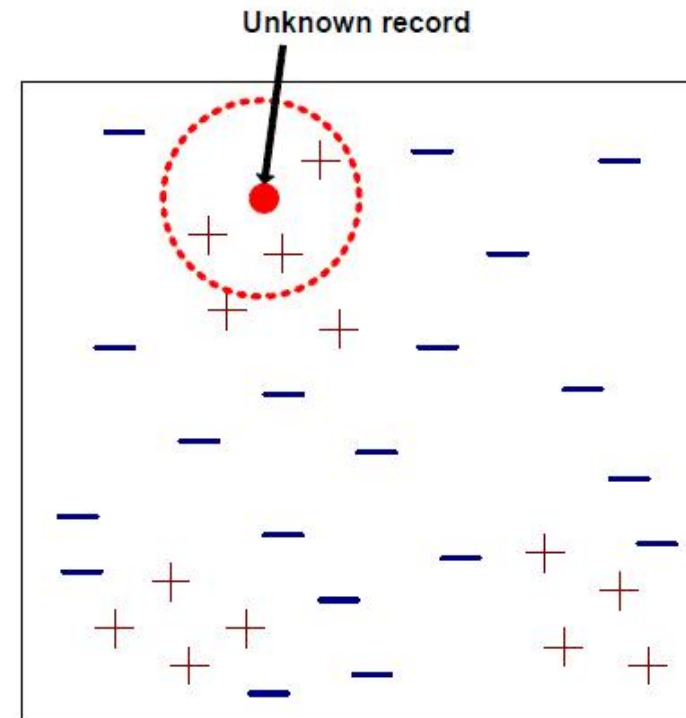
- Probabilistic & Geometric & Template matching
 - Geometric: Construct decision boundaries that divide feature space into classes
 - k-NN/ Nearest Mean (NM): geometrical distance between feature vectors of from different classes
 - Support Vector Machine (SVM): construct boundaries maximizing the margins between nearest features relative to two distinct classes

k-NN

- k Nearest Neighbor
- The simplest learning algorithm
- Lazy Learning Classifier
 - Given training data, it does nothing (don't model) until test data is given
 - cf. Eager Learning Classifier: decision tree, rule-based classifier

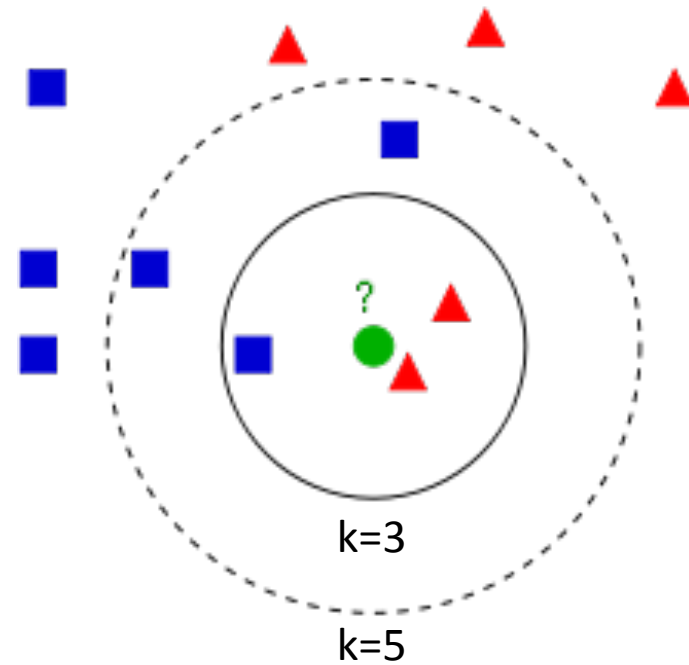
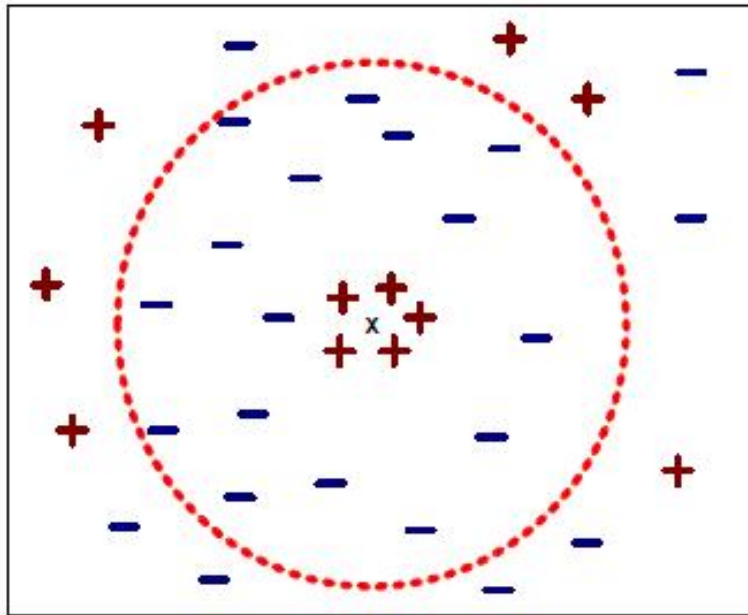
k-NN

- *“If it walks like a duck, quacks like a duck, and looks like a duck, then it’s probably a duck”*
- Plot each training data in space
- Given test data, compute the k nearest training data
- Test data is classified to the same class of majority of k-NN



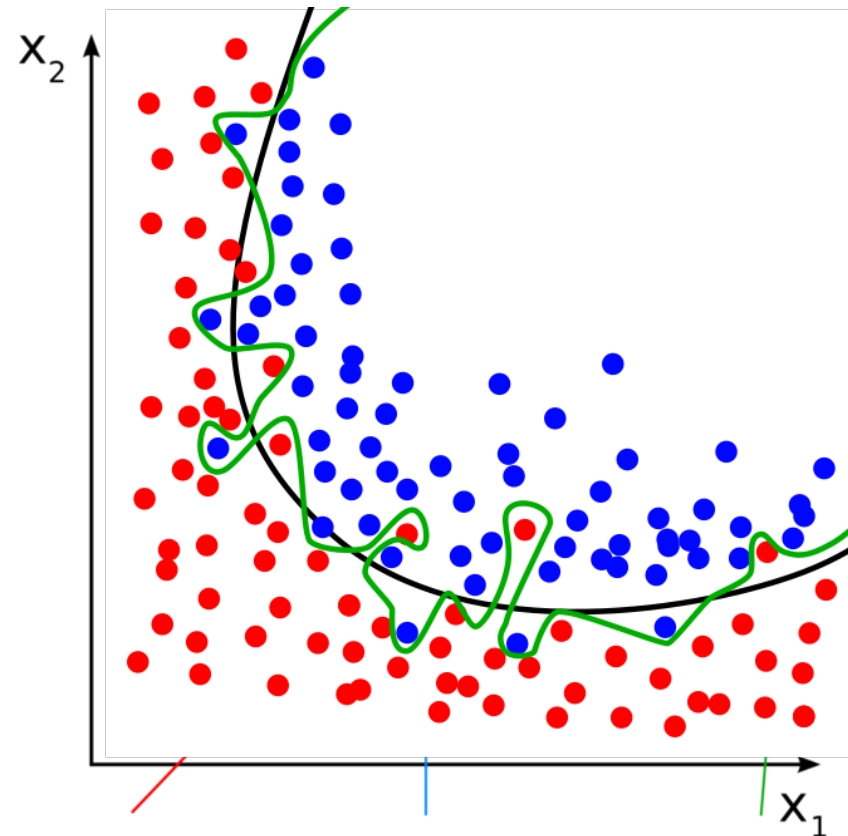
kNN: Choosing K

- Not too small, not too large



SVM(Support Vector Machine)

- Binary classification
- Find a hyperplane that separates two classes
- (evenly) maximize the distance to the nearest training data of any class (margin)



Type of Classifiers

- Probabilistic & Geometric & Template matching
 - Probabilistic: feature vector \mathbf{x} is classified to class C_{i^*} if class-conditional PDF $p(\mathbf{x}/C_i)$ is maximized for $i=1, \dots, C$
 - Optimal Bayesian classifier
 - Since class-conditional pdf is unknown, use suboptimal
 - naïve Bayesian, Logistic, Parzen, Gaussian Mixture Model (GMM)

Sequential Classifiers

- So far, single-frame classifiers
- Now, Sequential classifier
 - Exploit decisions made in the past
 - Composite activity is a chain of primitive activities
- Model a composite activity as a first-order Markov chain